

# Artificial Intelligence

A Modern Approach

*Fourth Edition*

*Global Edition*



**PEARSON SERIES  
IN ARTIFICIAL INTELLIGENCE**

*Stuart Russell and Peter Norvig, Editors*

FORSYTH & PONCE

GRAHAM

JURAFSKY & MARTIN

NEAPOLITAN

RUSSELL & NORVIG

*Computer Vision: A Modern Approach, 2nd ed.*

*ANSI Common Lisp*

*Speech and Language Processing, 2nd ed.*

*Learning Bayesian Networks*

*Artificial Intelligence: A Modern Approach, 4th ed.*

# Artificial Intelligence

A Modern Approach

*Fourth Edition*

*Global Edition*

Stuart J. Russell and Peter Norvig

*Contributing writers:*

Ming-Wei Chang

Jacob Devlin

Anca Dragan

David Forsyth

Ian Goodfellow

Jitendra M. Malik

Vikash Mansinghka

Judea Pearl

Michael Wooldridge



Cover Image credits: Alan Turing: Science History Images/Alamy Stock Photo; Statue of Aristotle: Panos Karas/Shutterstock; Ada Lovelace – Pictorial Press Ltd/Alamy Stock Photo; Autonomous cars: Andrey Suslov/Shutterstock; Atlas Robot: Boston Dynamics, Inc.; Berkeley Campanile and Golden Gate Bridge: Ben Chu/Shutterstock; Background ghosted nodes: Eugene Sergeev/Alamy Stock Photo; Chess board with chess figure: Titania/Shutterstock; Mars Rover: Stocktrek Images, Inc./Alamy Stock Photo; Kasparov: KATHY WILLENS/AP Images

*Pearson Education Limited*

KAO Two  
KAO Park  
Hockham Way  
Harlow  
CM17 9SR  
United Kingdom

and Associated Companies throughout the world

Visit us on the World Wide Web at: [www.pearsonglobaleditions.com](http://www.pearsonglobaleditions.com)

Please contact <https://support.pearson.com/getsupport/s/contactsupport> with any queries on this content

© Pearson Education Limited 2022

The rights of Stuart Russell and Peter Norvig to be identified as the authors of this work have been asserted by them in accordance with the Copyright, Designs and Patents Act 1988.

*Authorized adaptation from the United States edition, entitled Artificial Intelligence: A Modern Approach, 4th Edition, ISBN 978-0-13-461099-3 by Stuart J. Russell and Peter Norvig, published by Pearson Education © 2021.*

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without either the prior written permission of the publisher or a license permitting restricted copying in the United Kingdom issued by the Copyright Licensing Agency Ltd, Saffron House, 6–10 Kirby Street, London EC1N 8TS. For information regarding permissions, request forms and the appropriate contacts within the Pearson Education Global Rights & Permissions department, please visit [www.pearsoned.com/permissions/](http://www.pearsoned.com/permissions/).

PEARSON, ALWAYS LEARNING, and MYLAB are exclusive trademarks in the U.S. and/or other countries owned by Pearson Education, Inc. or its affiliates.

Unless otherwise indicated herein, any third-party trademarks that may appear in this work are the property of their respective owners and any references to third-party trademarks, logos or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc. or its affiliates, authors, licensees or distributors.

**ISBN 10:** 1-292-40113-3

**ISBN 13:** 978-1-292-40113-3

### **British Library Cataloguing-in-Publication Data**

A catalogue record for this book is available from the British Library

1            22

Typeset by SPi Global  
Printed and bound by

*For Loy, Gordon, Lucy, George, and Isaac — S.J.R.*

*For Kris, Isabella, and Juliet — P.N.*



# Preface

**Artificial Intelligence** (AI) is a big field, and this is a big book. We have tried to explore the full breadth of the field, which encompasses logic, probability, and continuous mathematics; perception, reasoning, learning, and action; fairness, trust, social good, and safety; and applications that range from microelectronic devices to robotic planetary explorers to online services with billions of users.

The subtitle of this book is “A Modern Approach.” That means we have chosen to tell the story from a current perspective. We synthesize what is now known into a common framework, recasting early work using the ideas and terminology that are prevalent today. We apologize to those whose subfields are, as a result, less recognizable.

## New to this edition

This edition reflects the changes in AI since the last edition in 2010:

- We focus more on machine learning rather than hand-crafted knowledge engineering, due to the increased availability of data, computing resources, and new algorithms.
- Deep learning, probabilistic programming, and multiagent systems receive expanded coverage, each with their own chapter.
- The coverage of natural language understanding, robotics, and computer vision has been revised to reflect the impact of deep learning.
- The robotics chapter now includes robots that interact with humans and the application of reinforcement learning to robotics.
- Previously we defined the goal of AI as creating systems that try to maximize expected utility, where the specific utility information—the objective—is supplied by the human designers of the system. Now we no longer assume that the objective is fixed and known by the AI system; instead, the system may be uncertain about the true objectives of the humans on whose behalf it operates. It must learn what to maximize and must function appropriately even while uncertain about the objective.
- We increase coverage of the impact of AI on society, including the vital issues of ethics, fairness, trust, and safety.
- We have moved the exercises from the end of each chapter to an online site. This allows us to continuously add to, update, and improve the exercises, to meet the needs of instructors and to reflect advances in the field and in AI-related software tools.
- Overall, about 25% of the material in the book is brand new. The remaining 75% has been largely rewritten to present a more unified picture of the field. 22% of the citations in this edition are to works published after 2010.

## Overview of the book

The main unifying theme is the idea of an **intelligent agent**. We define AI as the study of agents that receive percepts from the environment and perform actions. Each such agent implements a function that maps percept sequences to actions, and we cover different ways to represent these functions, such as reactive agents, real-time planners, decision-theoretic

systems, and deep learning systems. We emphasize learning both as a construction method for competent systems and as a way of extending the reach of the designer into unknown environments. We treat robotics and vision not as independently defined problems, but as occurring in the service of achieving goals. We stress the importance of the task environment in determining the appropriate agent design.

Our primary aim is to convey the *ideas* that have emerged over the past seventy years of AI research and the past two millennia of related work. We have tried to avoid excessive formality in the presentation of these ideas, while retaining precision. We have included mathematical formulas and pseudocode algorithms to make the key ideas concrete; mathematical concepts and notation are described in Appendix A and our pseudocode is described in Appendix B.

This book is primarily intended for use in an undergraduate course or course sequence. The book has 29 chapters, each requiring about a week's worth of lectures, so working through the whole book requires a two-semester sequence. A one-semester course can use selected chapters to suit the interests of the instructor and students. The book can also be used in a graduate-level course (perhaps with the addition of some of the primary sources suggested in the bibliographical notes), or for self-study or as a reference.

Throughout the book, *important points* are marked with a triangle icon in the margin. Wherever a new **term** is defined, it is also noted in the margin. Subsequent significant uses of the **term** are in bold, but not in the margin. We have included a comprehensive index and an extensive bibliography.

The only prerequisite is familiarity with basic concepts of computer science (algorithms, data structures, complexity) at a sophomore level. Freshman calculus and linear algebra are useful for some of the topics.

## Online resources

Online resources are available through [pearsonglobaleditions.com](http://pearsonglobaleditions.com). There you will find:

- Exercises, programming projects, and research projects. These are no longer at the end of each chapter; they are online only. Within the book, we refer to an online exercise with a name like “Exercise 6.NARY.” Instructions on the Web site allow you to find exercises by name or by topic.
- Implementations of the algorithms in the book in Python, Java, and other programming languages.
- Supplementary material and links for students and instructors.
- Instructions on how to report errors in the book in the likely event that some exist.

## Book cover

The cover depicts the final position from the decisive game 6 of the 1997 chess match in which the program Deep Blue defeated Garry Kasparov (playing Black), making this the first time a computer had beaten a world champion in a chess match. Kasparov is shown at the top. To his right is a pivotal position from the second game of the historic Go match between former world champion Lee Sedol and DeepMind's ALPHAGO program. Move 37 by ALPHAGO violated centuries of Go orthodoxy and was immediately seen by human experts



as an embarrassing mistake, but it turned out to be a winning move. At top left is an Atlas humanoid robot built by Boston Dynamics. A depiction of a self-driving car sensing its environment appears between Ada Lovelace, the world's first computer programmer, and Alan Turing, whose fundamental work defined artificial intelligence. At the bottom of the chess board are a Mars Exploration Rover robot and a statue of Aristotle, who pioneered the study of logic; his planning algorithm from *De Motu Animalium* appears behind the authors' names. Behind the chess board is a probabilistic programming model used by the UN Comprehensive Nuclear-Test-Ban Treaty Organization for detecting nuclear explosions from seismic signals.

## Acknowledgments

It takes a global village to make a book. Over 600 people read parts of the book and made suggestions for improvement. The complete list is at [pearsonglobaleditions.com](http://pearsonglobaleditions.com); we are grateful to all of them. We have space here to mention only a few especially important contributors. First the contributing writers:

- Judea Pearl (Section 13.5, Causal Networks);
- Michael Wooldridge (Chapter 17, Multiagent Decision Making);
- Vikash Mansinghka (Section 18.4, Programs as Probability Models);
- Ian Goodfellow (Chapter 22, Deep Learning);
- Jacob Devlin and Mei-Wing Chang (Chapter 25, Deep Learning for Natural Language Processing);
- Anca Dragan (Chapter 26, Robotics);
- Jitendra Malik and David Forsyth (Chapter 27, Computer Vision).

Then some key roles:

- Cynthia Yeung and Malika Cantor (project management);
- Julie Sussman and Tom Galloway (copyediting and writing suggestions);
- Omari Stephens (illustrations);
- Tracy Johnson (editor);
- Erin Ault and Rose Kernan (cover and color conversion);
- Nalin Chhibber, Sam Goto, Raymond de Lacaze, Ravi Mohan, Ciaran O'Reilly, Amit Patel, Dragomir Radiv, and Samagra Sharma (online code development and mentoring);
- Google Summer of Code students (online code development).

**Stuart would like to thank** his wife, Loy Sheflott, for her endless patience and boundless wisdom. He hopes that Gordon, Lucy, George, and Isaac will soon be reading this book after they have forgiven him for working so long on it. RUGS (Russell's Unusual Group of Students) have been unusually helpful, as always.

**Peter would like to thank** his parents (Torsten and Gerda) for getting him started, and his wife (Kris), children (Bella and Juliet), colleagues, boss, and friends for encouraging and tolerating him through the long hours of writing and rewriting.

## About the Authors

**Stuart Russell** was born in 1962 in Portsmouth, England. He received his B.A. with first-class honours in physics from Oxford University in 1982, and his Ph.D. in computer science from Stanford in 1986. He then joined the faculty of the University of California at Berkeley, where he is a professor and former chair of computer science, director of the Center for Human-Compatible AI, and holder of the Smith–Zadeh Chair in Engineering. In 1990, he received the Presidential Young Investigator Award of the National Science Foundation, and in 1995 he was cowinner of the Computers and Thought Award. He is a Fellow of the American Association for Artificial Intelligence, the Association for Computing Machinery, and the American Association for the Advancement of Science, an Honorary Fellow of Wadham College, Oxford, and an Andrew Carnegie Fellow. He held the Chaire Blaise Pascal in Paris from 2012 to 2014. He has published over 300 papers on a wide range of topics in artificial intelligence. His other books include *The Use of Knowledge in Analogy and Induction*, *Do the Right Thing: Studies in Limited Rationality* (with Eric Wefald), and *Human Compatible: Artificial Intelligence and the Problem of Control*.

**Peter Norvig** is currently a Director of Research at Google, Inc., and was previously the director responsible for the core Web search algorithms. He co-taught an online AI class that signed up 160,000 students, helping to kick off the current round of massive open online classes. He was head of the Computational Sciences Division at NASA Ames Research Center, overseeing research and development in artificial intelligence and robotics. He received a B.S. in applied mathematics from Brown University and a Ph.D. in computer science from Berkeley. He has been a professor at the University of Southern California and a faculty member at Berkeley and Stanford. He is a Fellow of the American Association for Artificial Intelligence, the Association for Computing Machinery, the American Academy of Arts and Sciences, and the California Academy of Science. His other books are *Paradigms of AI Programming: Case Studies in Common Lisp*, *VerbMobil: A Translation System for Face-to-Face Dialog*, and *Intelligent Help Systems for UNIX*.

The two authors shared the inaugural AAAI/EAAI Outstanding Educator award in 2016.

# Contents

## I Artificial Intelligence

<b>1 Introduction</b>	<b>19</b>
1.1 What Is AI? . . . . .	19
1.2 The Foundations of Artificial Intelligence . . . . .	23
1.3 The History of Artificial Intelligence . . . . .	35
1.4 The State of the Art . . . . .	45
1.5 Risks and Benefits of AI . . . . .	49
Summary . . . . .	52
Bibliographical and Historical Notes . . . . .	53
<b>2 Intelligent Agents</b>	<b>54</b>
2.1 Agents and Environments . . . . .	54
2.2 Good Behavior: The Concept of Rationality . . . . .	57
2.3 The Nature of Environments . . . . .	60
2.4 The Structure of Agents . . . . .	65
Summary . . . . .	78
Bibliographical and Historical Notes . . . . .	78

## II Problem-solving

<b>3 Solving Problems by Searching</b>	<b>81</b>
3.1 Problem-Solving Agents . . . . .	81
3.2 Example Problems . . . . .	84
3.3 Search Algorithms . . . . .	89
3.4 Uninformed Search Strategies . . . . .	94
3.5 Informed (Heuristic) Search Strategies . . . . .	102
3.6 Heuristic Functions . . . . .	115
Summary . . . . .	122
Bibliographical and Historical Notes . . . . .	124
<b>4 Search in Complex Environments</b>	<b>128</b>
4.1 Local Search and Optimization Problems . . . . .	128
4.2 Local Search in Continuous Spaces . . . . .	137
4.3 Search with Nondeterministic Actions . . . . .	140
4.4 Search in Partially Observable Environments . . . . .	144
4.5 Online Search Agents and Unknown Environments . . . . .	152
Summary . . . . .	159
Bibliographical and Historical Notes . . . . .	160
<b>5 Constraint Satisfaction Problems</b>	<b>164</b>
5.1 Defining Constraint Satisfaction Problems . . . . .	164
5.2 Constraint Propagation: Inference in CSPs . . . . .	169

5.3	Backtracking Search for CSPs . . . . .	175
5.4	Local Search for CSPs . . . . .	181
5.5	The Structure of Problems . . . . .	183
	Summary . . . . .	187
	Bibliographical and Historical Notes . . . . .	188
<b>6</b>	<b>Adversarial Search and Games</b>	<b>192</b>
6.1	Game Theory . . . . .	192
6.2	Optimal Decisions in Games . . . . .	194
6.3	Heuristic Alpha–Beta Tree Search . . . . .	202
6.4	Monte Carlo Tree Search . . . . .	207
6.5	Stochastic Games . . . . .	210
6.6	Partially Observable Games . . . . .	214
6.7	Limitations of Game Search Algorithms . . . . .	219
	Summary . . . . .	220
	Bibliographical and Historical Notes . . . . .	221
 <b>III Knowledge, reasoning, and planning</b>		
<b>7</b>	<b>Logical Agents</b>	<b>226</b>
7.1	Knowledge-Based Agents . . . . .	227
7.2	The Wumpus World . . . . .	228
7.3	Logic . . . . .	232
7.4	Propositional Logic: A Very Simple Logic . . . . .	235
7.5	Propositional Theorem Proving . . . . .	240
7.6	Effective Propositional Model Checking . . . . .	250
7.7	Agents Based on Propositional Logic . . . . .	255
	Summary . . . . .	264
	Bibliographical and Historical Notes . . . . .	265
<b>8</b>	<b>First-Order Logic</b>	<b>269</b>
8.1	Representation Revisited . . . . .	269
8.2	Syntax and Semantics of First-Order Logic . . . . .	274
8.3	Using First-Order Logic . . . . .	283
8.4	Knowledge Engineering in First-Order Logic . . . . .	289
	Summary . . . . .	295
	Bibliographical and Historical Notes . . . . .	296
<b>9</b>	<b>Inference in First-Order Logic</b>	<b>298</b>
9.1	Propositional vs. First-Order Inference . . . . .	298
9.2	Unification and First-Order Inference . . . . .	300
9.3	Forward Chaining . . . . .	304
9.4	Backward Chaining . . . . .	311
9.5	Resolution . . . . .	316
	Summary . . . . .	327
	Bibliographical and Historical Notes . . . . .	328

<b>10 Knowledge Representation</b>	<b>332</b>
10.1 Ontological Engineering . . . . .	332
10.2 Categories and Objects . . . . .	335
10.3 Events . . . . .	340
10.4 Mental Objects and Modal Logic . . . . .	344
10.5 Reasoning Systems for Categories . . . . .	347
10.6 Reasoning with Default Information . . . . .	351
Summary . . . . .	355
Bibliographical and Historical Notes . . . . .	356
<b>11 Automated Planning</b>	<b>362</b>
11.1 Definition of Classical Planning . . . . .	362
11.2 Algorithms for Classical Planning . . . . .	366
11.3 Heuristics for Planning . . . . .	371
11.4 Hierarchical Planning . . . . .	374
11.5 Planning and Acting in Nondeterministic Domains . . . . .	383
11.6 Time, Schedules, and Resources . . . . .	392
11.7 Analysis of Planning Approaches . . . . .	396
Summary . . . . .	397
Bibliographical and Historical Notes . . . . .	398
<b>IV Uncertain knowledge and reasoning</b>	
<b>12 Quantifying Uncertainty</b>	<b>403</b>
12.1 Acting under Uncertainty . . . . .	403
12.2 Basic Probability Notation . . . . .	406
12.3 Inference Using Full Joint Distributions . . . . .	413
12.4 Independence . . . . .	415
12.5 Bayes' Rule and Its Use . . . . .	417
12.6 Naive Bayes Models . . . . .	420
12.7 The Wumpus World Revisited . . . . .	422
Summary . . . . .	425
Bibliographical and Historical Notes . . . . .	426
<b>13 Probabilistic Reasoning</b>	<b>430</b>
13.1 Representing Knowledge in an Uncertain Domain . . . . .	430
13.2 The Semantics of Bayesian Networks . . . . .	432
13.3 Exact Inference in Bayesian Networks . . . . .	445
13.4 Approximate Inference for Bayesian Networks . . . . .	453
13.5 Causal Networks . . . . .	467
Summary . . . . .	471
Bibliographical and Historical Notes . . . . .	472
<b>14 Probabilistic Reasoning over Time</b>	<b>479</b>
14.1 Time and Uncertainty . . . . .	479
14.2 Inference in Temporal Models . . . . .	483

14.3	Hidden Markov Models . . . . .	491
14.4	Kalman Filters . . . . .	497
14.5	Dynamic Bayesian Networks . . . . .	503
	Summary . . . . .	514
	Bibliographical and Historical Notes . . . . .	515
<b>15</b>	<b>Making Simple Decisions</b>	<b>518</b>
15.1	Combining Beliefs and Desires under Uncertainty . . . . .	518
15.2	The Basis of Utility Theory . . . . .	519
15.3	Utility Functions . . . . .	522
15.4	Multiattribute Utility Functions . . . . .	530
15.5	Decision Networks . . . . .	534
15.6	The Value of Information . . . . .	537
15.7	Unknown Preferences . . . . .	543
	Summary . . . . .	547
	Bibliographical and Historical Notes . . . . .	547
<b>16</b>	<b>Making Complex Decisions</b>	<b>552</b>
16.1	Sequential Decision Problems . . . . .	552
16.2	Algorithms for MDPs . . . . .	562
16.3	Bandit Problems . . . . .	571
16.4	Partially Observable MDPs . . . . .	578
16.5	Algorithms for Solving POMDPs . . . . .	580
	Summary . . . . .	585
	Bibliographical and Historical Notes . . . . .	586
<b>17</b>	<b>Multiagent Decision Making</b>	<b>589</b>
17.1	Properties of Multiagent Environments . . . . .	589
17.2	Non-Cooperative Game Theory . . . . .	595
17.3	Cooperative Game Theory . . . . .	616
17.4	Making Collective Decisions . . . . .	622
	Summary . . . . .	635
	Bibliographical and Historical Notes . . . . .	636
<b>18</b>	<b>Probabilistic Programming</b>	<b>641</b>
18.1	Relational Probability Models . . . . .	642
18.2	Open-Universe Probability Models . . . . .	648
18.3	Keeping Track of a Complex World . . . . .	655
18.4	Programs as Probability Models . . . . .	660
	Summary . . . . .	664
	Bibliographical and Historical Notes . . . . .	665
<b>V</b>	<b>Machine Learning</b>	
<b>19</b>	<b>Learning from Examples</b>	<b>669</b>
19.1	Forms of Learning . . . . .	669

19.2	Supervised Learning . . . . .	671
19.3	Learning Decision Trees . . . . .	675
19.4	Model Selection and Optimization . . . . .	683
19.5	The Theory of Learning . . . . .	690
19.6	Linear Regression and Classification . . . . .	694
19.7	Nonparametric Models . . . . .	704
19.8	Ensemble Learning . . . . .	714
19.9	Developing Machine Learning Systems . . . . .	722
	Summary . . . . .	732
	Bibliographical and Historical Notes . . . . .	733
<b>20</b>	<b>Knowledge in Learning</b>	<b>739</b>
20.1	A Logical Formulation of Learning . . . . .	739
20.2	Knowledge in Learning . . . . .	747
20.3	Explanation-Based Learning . . . . .	750
20.4	Learning Using Relevance Information . . . . .	754
20.5	Inductive Logic Programming . . . . .	758
	Summary . . . . .	767
	Bibliographical and Historical Notes . . . . .	768
<b>21</b>	<b>Learning Probabilistic Models</b>	<b>772</b>
21.1	Statistical Learning . . . . .	772
21.2	Learning with Complete Data . . . . .	775
21.3	Learning with Hidden Variables: The EM Algorithm . . . . .	788
	Summary . . . . .	797
	Bibliographical and Historical Notes . . . . .	798
<b>22</b>	<b>Deep Learning</b>	<b>801</b>
22.1	Simple Feedforward Networks . . . . .	802
22.2	Computation Graphs for Deep Learning . . . . .	807
22.3	Convolutional Networks . . . . .	811
22.4	Learning Algorithms . . . . .	816
22.5	Generalization . . . . .	819
22.6	Recurrent Neural Networks . . . . .	823
22.7	Unsupervised Learning and Transfer Learning . . . . .	826
22.8	Applications . . . . .	833
	Summary . . . . .	835
	Bibliographical and Historical Notes . . . . .	836
<b>23</b>	<b>Reinforcement Learning</b>	<b>840</b>
23.1	Learning from Rewards . . . . .	840
23.2	Passive Reinforcement Learning . . . . .	842
23.3	Active Reinforcement Learning . . . . .	848
23.4	Generalization in Reinforcement Learning . . . . .	854
23.5	Policy Search . . . . .	861
23.6	Apprenticeship and Inverse Reinforcement Learning . . . . .	863

23.7 Applications of Reinforcement Learning . . . . .	866
Summary . . . . .	869
Bibliographical and Historical Notes . . . . .	870

## VI Communicating, perceiving, and acting

<b>24 Natural Language Processing</b>	<b>874</b>
24.1 Language Models . . . . .	874
24.2 Grammar . . . . .	884
24.3 Parsing . . . . .	886
24.4 Augmented Grammars . . . . .	892
24.5 Complications of Real Natural Language . . . . .	896
24.6 Natural Language Tasks . . . . .	900
Summary . . . . .	901
Bibliographical and Historical Notes . . . . .	902
<b>25 Deep Learning for Natural Language Processing</b>	<b>907</b>
25.1 Word Embeddings . . . . .	907
25.2 Recurrent Neural Networks for NLP . . . . .	911
25.3 Sequence-to-Sequence Models . . . . .	915
25.4 The Transformer Architecture . . . . .	919
25.5 Pretraining and Transfer Learning . . . . .	922
25.6 State of the art . . . . .	926
Summary . . . . .	929
Bibliographical and Historical Notes . . . . .	929
<b>26 Robotics</b>	<b>932</b>
26.1 Robots . . . . .	932
26.2 Robot Hardware . . . . .	933
26.3 What kind of problem is robotics solving? . . . . .	937
26.4 Robotic Perception . . . . .	938
26.5 Planning and Control . . . . .	945
26.6 Planning Uncertain Movements . . . . .	963
26.7 Reinforcement Learning in Robotics . . . . .	965
26.8 Humans and Robots . . . . .	968
26.9 Alternative Robotic Frameworks . . . . .	975
26.10 Application Domains . . . . .	978
Summary . . . . .	981
Bibliographical and Historical Notes . . . . .	982
<b>27 Computer Vision</b>	<b>988</b>
27.1 Introduction . . . . .	988
27.2 Image Formation . . . . .	989
27.3 Simple Image Features . . . . .	995
27.4 Classifying Images . . . . .	1002
27.5 Detecting Objects . . . . .	1006



27.6	The 3D World . . . . .	1008
27.7	Using Computer Vision . . . . .	1013
	Summary . . . . .	1026
	Bibliographical and Historical Notes . . . . .	1027

## VII Conclusions

<b>28</b>	<b>Philosophy, Ethics, and Safety of AI</b>	<b>1032</b>
28.1	The Limits of AI . . . . .	1032
28.2	Can Machines Really Think? . . . . .	1035
28.3	The Ethics of AI . . . . .	1037
	Summary . . . . .	1056
	Bibliographical and Historical Notes . . . . .	1057
<b>29</b>	<b>The Future of AI</b>	<b>1063</b>
29.1	AI Components . . . . .	1063
29.2	AI Architectures . . . . .	1069
<b>A</b>	<b>Mathematical Background</b>	<b>1074</b>
A.1	Complexity Analysis and $O()$ Notation . . . . .	1074
A.2	Vectors, Matrices, and Linear Algebra . . . . .	1076
A.3	Probability Distributions . . . . .	1078
	Bibliographical and Historical Notes . . . . .	1080
<b>B</b>	<b>Notes on Languages and Algorithms</b>	<b>1081</b>
B.1	Defining Languages with Backus–Naur Form (BNF) . . . . .	1081
B.2	Describing Algorithms with Pseudocode . . . . .	1082
B.3	Online Supplemental Material . . . . .	1083
	<b>Bibliography</b>	<b>1084</b>
	<b>Index</b>	<b>1119</b>

